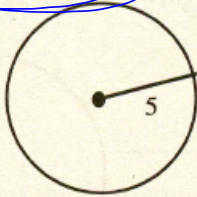


Radian Measure

Warm-Up #1

- i) What is the formula for the circumference of a circle of radius r ?
- ii) Find the circumference of the following circle. Leave your answer as an exact value in terms of π .



$$C = 2\pi(5) = 10\pi$$

$$C = 2\pi r \quad (\pi d)$$

Warm-Up #2

In all previous work with angular measure we have used **degree** measure.

One degree is defined as $\frac{1}{360}$ of a revolution.

In order to simplify some of the calculations involved in trigonometry and calculus, mathematicians use an alternative angular measure - **radian** measure.

The Radian Measure of an Angle

The radian measure of an angle is a ratio that compares the length of an arc of a circle to the radius of the circle, i.e.

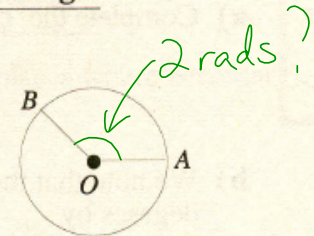
$$\text{measure of an angle in radians} = \frac{\text{length of arc subtending the angle}}{\text{length of radius}}$$

- The radian measure of $\angle AOB$ is given by the

ratio $\frac{\text{arc } AB}{\text{radius } OA}$ (see diagram 1)

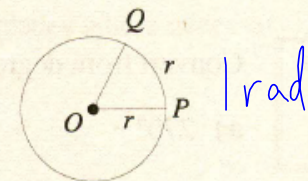
$\theta_{\text{rad}} = \frac{\text{arc}}{\text{radius}}$

Diagram 1



- One radian is the measure of the angle at the centre of a circle subtended by an arc equal in length to the radius of the circle (see diagram 2)

Diagram 2



$\angle POQ = 1 \text{ radian}$

- Use diagram 1 and the definition to estimate the radian measure of $\angle AOB$.

- Use diagram 2 to estimate the degree measure of $\angle POQ$.

Handwritten notes and calculations:

- $\frac{180^\circ}{\pi} \approx 60^\circ (57.3\dots)$
- $1 \text{ rad} = 60^\circ (57.3^\circ)$
- $\frac{\pi}{180} = \frac{1}{\dots}$ (crossed out)
- $\frac{\pi}{180} = 1 \times \frac{180}{\pi} \text{ rads} \rightarrow \text{deg}$
- $\pi = 180^\circ$
- $3.14 = 180^\circ$

Converting Between Degrees and Radians

Since an angle can be measured in degrees or radians, it is important in to be able to convert from one measure to the other.

Warm-Up #3

Consider a circle with a radius of r units. Complete the following:

a) i) one complete rotation in degrees is 360° .

ii) the arc length for one complete rotation is $2\pi r$
which is the circumference of the circle.

iii) the radian measure of an angle of 360° is 2π .

b) i) one-half rotation in degrees is 180° .

ii) the arc length for one-half rotation is πr .

iii) the radian measure of an angle of 180° is π .

- In mathematics, the symbol " $^\circ$ " following a number means the unit of angular measure is degrees.
- If there is no unit after the number, or there is the abbreviation "rad", or the word radians, then the unit is radians.
- For example, if you wish to write the sine ratio for a right angle, you must write $\sin 90^\circ$, and NOT $\sin 90$.



Radian \leftrightarrow Degree

pg. 175-179

#2-5, 17, 19, 24, 25, C1, C2, C5

$$\text{Rads} \rightarrow \text{Degrees} \quad \theta \times \frac{180^\circ}{\pi}$$

$$\text{Degrees} \rightarrow \text{Rads} \quad \theta \times \frac{\pi}{180^\circ}$$

Arc Length

pg. 175-179

#12-14, 16, 26, 27

$$\text{arc length} = \theta \times r$$

↑
in rads